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TURBULENT HEAT AND MOMENTUM TRANSFER IN A SQUARE DUCT WITH MODERATE PROPERTY VARIATIONS

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NOMENCLATURE

A, flow area;
D_h, hydraulic diameter, 4 (flow area)/wetted perimeter;
f, fanning friction factor;
$\dot{m}$, mass flow rate;
Nu, Nusselt number;
P, pressure;
Pr, Prandtl number;
Re, Reynolds number;
T, temperature;
V, bulk mean velocity;
x, distance downstream from thermal entry.

Greek symbols
ρ, density;
τ, shear stress;
μ, viscosity.

Subscripts
b, evaluated at bulk properties.

INTRODUCTION

RESULTS are presented for local heat transfer and friction coefficients for the turbulent flow of air in a vertical, nominally square duct. Heating rates were high enough to provide significant radial and axial variations in viscosity, conductivity and density. The variation of the transport properties is of primary interest here. The results should be of interest to present day designers of nuclear rockets, gas cooled reactors, and high performance heat exchangers.

Turbulent, variable-property, heat transfer has been studied for gas flow in circular tubes for some time. However, for non-circular ducts local results are available only for a nominally equilateral triangular duct [1]. These results indicate that the local friction factors are some 20 per cent higher than those for a circular tube under similar high heating rate conditions; while local heat-transfer coefficients are some 10 per cent lower than the circular tube results. Lowdermilk *et al.* [2] obtained measurements for air flow through a square duct and found that the results for *average* heat-transfer coefficients could be correlated to circular duct results when the properties were evaluated at mean film temperatures. The present results are for local friction and heat-transfer coefficients determined at each thermocouple and pressure tap.

The test section consisted of an extruded Inconel 600 tube, typical of that commercially available, with the nominal cross-sectional shape of a square. The nominal size was 0·125 in. outside wall length with a 0·015 in. wall thickness. Photographic analysis of the cross section showed a true hydraulic diameter of 0·095 in. and a ratio of the corner radius of curvature to hydraulic diameter of approximately 0·12.

The test section was constructed in the manner of Campbell and Perkins [1] with an isothermal hydrodynamic development length of 100 diameters. The experimental apparatus was substantially the same as that used in [1]. Corrections to the data were made for the fin effect of the thermocouples welded to the tube for the local heat loss from the electrically heated tube, and for the axial heat loss at the upper and lower electrodes. The computer program to reduce the data is described in [1] and uses air properties from NBS 564 [3].

Data taken on a previous test section with a square cross-section showed temperature differences, between the midwall and corner position, as measured with thermocouples attached to the outside of the wall, to be small compared to the wall to bulk temperature difference. Specifically under conditions corresponding to the present data, midwall to corner temperature differences of 14°F, at wall to bulk temperature differences of about 350°F, were noted. A maximum value of 25°F at a 450°F temperature difference was noted. In short peripheral variations were at most 5 per cent of the temperature differences. These maximum values occurred at the largest heat fluxes.

The peripheral conduction parameter of Eckert and Irvine [4], $k_w b/kD_h$ was about 100 in these tests. Here b is the wall thickness, k_w the wall thermal conductivity, and k the fluid thermal conductivity.

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The thermal boundary condition on the electrically heated tube approximated a constant heat flux. Since a larger heat loss occurred at the upper (hotter) end of the test section some reduction in the heat flux to the gas did occur. Three of the fifteen runs showed axial heat flux variations of greater than 15 per cent. In the worst case the heat flux at the upper end fell by 26 per cent from the value at the lower end of the test section. The wall temperature rose sharply at the lower electrode of the test section and then rose more slowly, typical of a constant heat flux duct. Consequently the wall to bulk temperature ratio was a function of axial position and appeared very much as shown in [5]. After an initial rise in the thermal entrance region its value decreased along the duct.

Since it is known that turbulent heat-transfer results are not greatly sensitive to the thermal boundary conditions the present results are considered to not be strongly effected by either the moderate peripheral temperature variations or the axial heat flux variations.

RESULTS

Figure 1 presents the local heat-transfer results for inlet Reynolds numbers from 21000 to 49000; maximum T_w/T_b to 2·13; and axial distances from $22 < x/D < 155$. The

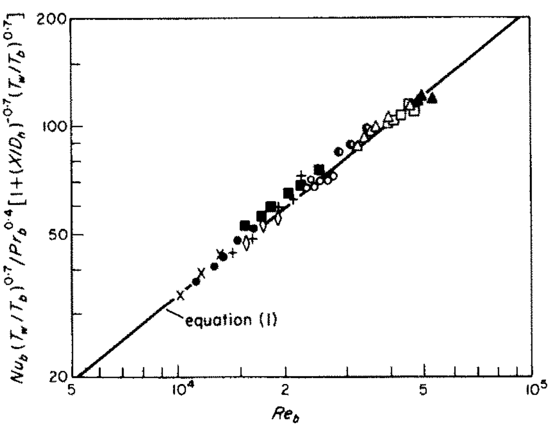

FIG. 1. Local heat-transfer results based on bulk properties. Different symbols represent different experimental runs

correlation achieved uses bulk properties and a wall-to-bulk temperature ratio correction to account for the effect of the variable properties and is given by the equation

$$Nu_b = 0{\cdot}021\,Re_b^{0.8}Pr_b^{0.4}(T_w/T_b)^{-0.7} \times [1 + (x/D_h)^{-0.7}(T_w/T_b)]. \quad (1)$$

This equation is the same as that found by Campbell and Perkins [1] for local heat transfer coefficients in a nominally equilateral triangular duct. The results are some 10 per cent below corresponding circular tube results [5, 6]. The entry correction was applied to an x/D_h of fifty, after that point the term in brackets was neglected in correlating the data.

Downstream friction results are presented in Fig. 2. The local (fanning) friction factor is defined as

$$f = \tau_w/(\rho V^2/2g_c). \quad (2)$$

The wall shear is determined experimentally from the expression

$$\tau_w = -(D_h/4)\frac{\mathrm{d}}{\mathrm{d}x}[P + \rho V^2/g_c]. \quad (3)$$

This result assumes the static pressure is constant across the flow and treats the momentum flux as one-dimensional.

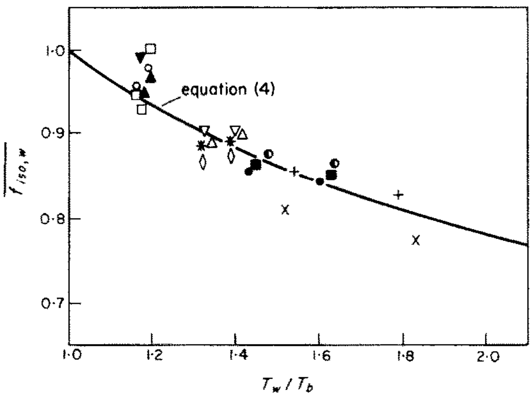

FIG. 2. Local friction coefficients based on bulk properties, normalized on the isothermal friction coefficients for the same wall Reynolds number. Different symbols represent different experimental runs.

Adiabatic friction results were obtained during the test runs to establish the tube characteristics to check for aging or oxidation effects. These results indicated good agreement with the Kármán–Nikuradse line in agreement with the adiabatic results presented by Hartnett *et al.* [7] for square and rectangular ducts. A relative roughness of 0·001 was indicated both by the friction results and by photographic analysis. The friction results with heating were obtained for $22 < x/D < 137$ and are some 20 per cent higher than those for the circular tube under similar heating conditions. The correlation is the same as that found in [1] for the equilateral triangular duct and is

$$f/f_{\mathrm{iso},w} = [T_w/T_b]^{-0.35} \quad (4)$$

The friction factor is based on one-dimensional bulk properties but is normalized by the present experimental friction factor evaluated at the heated modified *wall* Reynolds number in the manner of Perkins and Worsøe-Schmidt [5]. The wall Reynolds number is defined as

$$Re_w = \frac{D_h \dot{m}}{A\mu_w} \frac{T_b}{T_w} \quad (5)$$

and for perfect gas becomes equivalent to

$$Re_w = \frac{V_b \rho_b D_h}{\mu_w} \frac{\rho_w}{\rho_b} = \frac{V_b \rho_w D_h}{\mu_w}.$$

For design purposes if one is satisfied with an uncertainty of 15 per cent both these friction results with significant heating and the results for the equilateral triangle [1] may be correlated, using a Reynolds number based on local bulk properties, directly on the constant property correlation such as the Kármán–Nakuradse line. The effect of heating on the friction coefficient is therefore small, just as in the circular tube case [5, 6].

CONCLUSIONS

Heat-transfer results with significant heating in a square vertical duct are in good agreement with those presented previously for an equilateral triangular duct and are some 10 per cent below the circular tube results. Friction results are also in agreement with the triangular results and are some 20 per cent higher than the circular tube results at the same heating conditions.

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